Bounds for Forward Recurrence Times in Point Processes with Dependent Inter-Event Times

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Abstract. The forward recurrence time–the delay from an observation epoch to the next event–is fundamental in renewal and reliability models, queueing arrival streams, and self-exciting point processes. We consider an adapted simple non-explosive point process with possibly dependent, non-identically distributed inter-event times. Combining a conditional upper bound on each successive inter-event tail with deterministic domination of the mean counting measure of event epochs, we derive finite-time bounds for $\mathbb{P}(W_t > x)$ and $\mathbb{E}f(W_t)$ for non-negative, non-decreasing, locally absolutely continuous f. The comparison is exact for non-homogeneous Poisson processes. Homogeneous conditions yield uniform and asymptotic Lorden-type positive-moment bounds and exponential-moment estimates. An i.i.d. renewal counterexample shows that gap-tail control alone yields no class-uniform first-moment bound. A bounded-hazard corollary covers reliability and queueing streams. A linear Hawkes benchmark and a quadratic-feedback boundary example illustrate model-specific sharpening and the independent role of non-explosion.

Keywords: forward recurrence time · dependent inter-event times · Lorden-type bound · renewal process · reliability · queueing · Hawkes process