Opinion Dynamics in a Principal-Agent Network under Incomplete Information: An Adaptive Control Approach

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This study builds upon the classical DeGroot model of opinion dynamics, extending it by introducing a hierarchical network structure and an external control problem. We consider a game in a social network consisting of one leader (principal) and a group of identical regular agents. Each agent is characterized by levels of trust in themselves and, separately, in the principal. An external player intervenes in this network, aiming to steer the network's final consensus toward a specific target value with minimal cost by exerting control directly over the principal. This paper introduces and analyzes three models of control strategies. The first model assumes constant control under complete information about the network's trust parameters. The second model describes time-dependent optimal control, analytically derived by solving the Bellman equation. The third model, which is closest to real-world conditions, focuses on adaptive control under incomplete information, where mutual trust parameters are treated as random variables with given distributions. Numerical simulation results demonstrate that applying optimal strategies allows the external player to effectively shift the system's stationary state toward the target value. Notably, the proposed approach to adaptive control under incomplete information achieves a consensus quality that is practically indistinguishable from that of the strategy applied in the complete information model.