A Production-Queueing-Inventory System with Server and Machine Degradation Following Coxian Distributions

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Abstract

We study a single-server production-queueing-inventory system in which both the service facility and the production machine are subject to progressive degradation over time. Customers arrive according to a Poisson process and request one unit of inventory. The inventory is managed under a strict (s, S) replenishment policy: production activates when the on-hand stock drops to s or below and continues item-by-item until the maximum capacity S is reached. Both the server and the production machine evolve through multiple degradation stages with stage-dependent exponential service and production rates.

The operational lifespans of both the server and the machine are modeled using Coxian distributions. This mathematical formulation explicitly assumes that equipment does not necessarily progress through all possible wear stages before failure. Instead, upon completing a given operational stage, the equipment either progresses to the next degradation stage or undergoes a replacement. Specifically, a server completing stage u will transition to stage $u + 1$ with probability p_u, or it will undergo an instantaneous replacement with probability $1 - p_u$, allowing it to immediately resume operation at its initial stage. Likewise, a production machine completing stage v either degrades to stage $v + 1$ with probability q_v or is instantaneously replaced with probability $1 - q_v$.

By formulating this dynamic environment as a Continuous-Time Markov Chain (CTMC) with a Level-Independent Quasi-Birth-Death (LIQBD) structure, the study analytically derives the system's stability condition. The steady-state probabilities are computed by applying the Matrix-Geometric Method.

Utilizing the stationary distribution, the authors formulate expressions for key system performance measures. These measures are aggregated into a total expected cost function, providing a rigorous mathematical foundation for the numerical optimization.