On an (s_1, K, s_2, S) Queueing Inventory System with Stochastic Centering, Reciprocal Bidirectional Stock Flows and Product-Form Solutions

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Abstract

This paper analyses a single server queueing inventory system with reciprocal bidirectional stock flows and dual threshold controls, in which customers (buyers and sellers) arrive according to a marked Poisson process with rates $p\lambda$ and $(1-p)\lambda$ respectively. A customer reveals his/her demand only at the epoch of service. He/She either buys an item or sells an item to the inventory pool. These are termed as type 1 and type 2 demands respectively. Service corresponding to type 1 and type 2 demands is exponentially distributed with rates μ_1 and μ_2 respectively. Here we introduce a new inventory control policy (s_1, K, s_2, S), where S is the maximum inventory capacity that the system can hold, s_2 is the upper threshold inventory level at which an inventory clearance order is placed, s_1 is the lower threshold inventory level at which an inventory replenishment order is placed. Both orders are placed to make the inventory level back to K, where $s_1 < K < s_2 < S$. The two lead times are exponentially distributed with rates β_1 and β_2 respectively. Surprisingly, the proposed model yields a product form solution while incorporating partial customer blocking at both zero and full inventory levels, thereby providing a novel contribution to the queueing inventory literature. Some important system performance measures are discussed and a system cost function is constructed in order to optimize the system parameters s_1, K, s_2 and S. Numerical illustrations are done to analyse how the system behaves with respect to changes in different parameters.

Highlights of the Work

- **Reciprocal Bidirectional Stock flows:** Moving beyond traditional unidirectional models, this system features dual channel inflows (replenishments and procurements) and dual channel outflows (sales and clearances).
- **Novel Control Policy:** To the best of our knowledge this work is the first to introduce and analyse the (s_1, K, s_2, S) inventory control policy, designed to anchor stock levels to a strategic target level K.

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- **Partial Blocking Policy:** A feature of this model is the partial blocking policy at the boundaries $i = 0$ and $i = S$, which allows the system to 'self-heal' through individual arrivals even while awaiting a policy driven bulk reset.
- **Product-Form Solution:** Another major contribution of this work is establishing that, under symmetric arrival and service rate conditions, the dynamics of this complex system decouple to yield an explicit product form solution.
- **Dual Exponential Lead Times:** The system incorporates dual lead time dynamics (replenishments and clearance) which are exponentially distributed with rates β_1 and β_2.
- **Service Epoch Revelation:** The model accounts for information latency by revealing customer intent, either to buy or to sell at the epoch of commencement of service.
- **Non-Standard Matrix Structure:** Unlike the classical inventory with replenishment through order where the replenishment rate appears in the last column of the matrices B_0 and A_1 in the infinitesimal generator $\mathcal{Q}$, in the present set up the replenishment and clearance rates appear in the intermediate position between s_1 and s_2.